

航空面齿轮磨削碟形砂轮修整方法及误差分析

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摘 要: 根据面齿轮专用磨齿机的机械结构,提出了偏置曲线修整方法,设计了砂轮修整装置,并运用 VERICUT 软件进行修整加工仿真.推导了含各类修整误差情况下面齿轮的齿面方程表达式,并运用 MATLAB 软件生成了齿面误差云图,为修整过程的误差补偿提供了理论参考.最后进行了磨削及检测试验,将检测结果与误差云图对比,分析可能产生这种误差的原因,并在修整过程中进行误差补偿,使齿面最大法向误差值减少了 31.14%,面齿轮齿形精度得到了提高.

关 键 词: 面齿轮; 渐开线; 砂轮修整; 误差分析; 误差补偿

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Dressing method and dressing error analysis of aviation face gear grinding wheel

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Abstract: According to the mechanical structure of a face gear grinding machine, an off-set curve dressing method and a grinding wheel dresser were designed, and simulation and analysis of the dressing process were performed with VERICUT. The equation of face-gear tooth profile with error caused by wheel dressing process was derived and the error cloud of face-gear tooth profile was created with MATLAB, providing theory references for the research on error compensation of dressing process. The grinding and testing experiments were carried out at last. By comparing the result with the error cloud, the reason was analyzed and the error in the dressing process was compensated; the maximum tooth profile normal error was reduced by 31.14%, and the precision of the tooth surface was improved.

Key words: face gear; involute; wheel dressing; error analysis; error compensation

面齿轮传动是一种新型交错轴传动方式,具有质量轻、结构紧凑、低振动及噪声、分流效果好等优势,在航空工业领域具有广阔的应用前

景^[1-8].国内对面齿轮的研究主要集中在南京航空航天大学^[9-10]、北京工业大学^[11]、西北工业大学^[12]和北京航空航天大学^[13-14]等.磨齿是面齿轮

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的主要精加工工艺,砂轮修整精度直接反映到磨齿齿面精度中.郭辉等研究了碟形砂轮安装误差对面齿轮齿面的影响规律^[15],彭先龙等研究了小齿轮修形对面齿轮啮合性能的影响规律^[16],Litvin 等研究了蜗杆砂轮安装误差对啮合区的影响规律^[17],Shih 等分析了五轴数控齿轮磨床加工误差并进行了补偿技术研究^[18].而在碟形砂轮修整误差对面齿轮齿面精度的影响规律方面的研究还不够深入.

本文以渐开线廓型的面齿轮磨削碟形砂轮为研究对象,提出了偏置曲线修整方法,并设计了相应的修整器结构,推导分析了带修整误差的面齿轮齿面方程,生成了误差云图,根据误差规律进行修整过程的误差补偿,并进行了修整及磨削加工的试验验证.

1 面齿轮齿面方程函数表达

面齿轮齿面是由渐开线砂轮经过复杂的空间展成运动形成的. $S_{s0}(x_{s0}, y_{s0}, z_{s0})$, $S_{20}(x_{20}, y_{20}, z_{20})$ 为固定坐标系,分别是砂轮和面齿轮的初始位置. $S_s(x_s, y_s, z_s)$ 为砂轮绕轴 z_s 以 ω_s 的角速度回转形成的动坐标系,某一时刻砂轮转角为 φ_s ; $S_2(x_2, y_2, z_2)$ 为面齿轮绕轴 z_2 以 ω_2 的角速度回转形成的动坐标系,相应时刻面齿轮转角为 φ_2 . 展成过程坐标系可由图 1 表达.

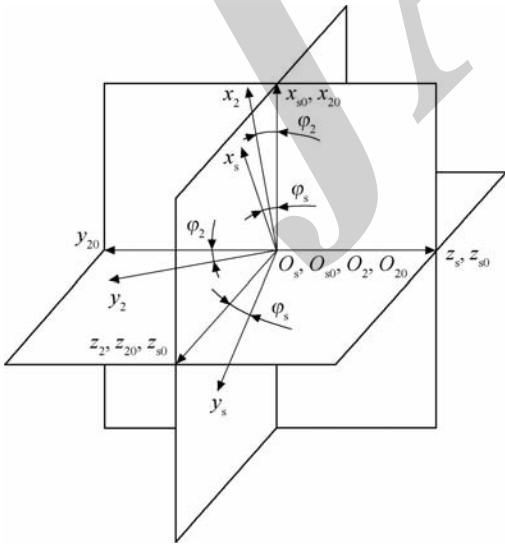


图 1 面齿轮展成过程坐标系

Fig. 1 Coordinate system of face gear generating process

面齿轮齿面方程为

$$\mathbf{r}_2(u_s, \theta_{ks}, \varphi_s) = \mathbf{M}_{2s}(\varphi_s) \mathbf{r}_s(u_s, \theta_{ks}) \quad (1)$$

坐标系 S_s 到 S_2 的坐标变换矩阵为

$$\mathbf{M}_{2s} =$$

$$\begin{bmatrix} \cos \varphi_2 \cos \varphi_s & -\cos \varphi_2 \sin \varphi_s & -\sin \varphi_2 & 0 \\ -\sin \varphi_2 \cos \varphi_s & \sin \varphi_2 \sin \varphi_s & -\cos \varphi_2 & 0 \\ \sin \varphi_s & \cos \varphi_s & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

砂轮渐开线方程

$$\mathbf{r}_s(u_s, \theta_{ks}) =$$

$$\begin{bmatrix} \pm r_{bs} [\sin(\theta_{os} + \theta_{ks}) - \theta_{ks} \cos(\theta_{os} + \theta_{ks})] \\ -r_{bs} [\cos(\theta_{os} + \theta_{ks}) + \theta_{ks} \sin(\theta_{os} + \theta_{ks})] \\ u_s \\ 1 \end{bmatrix} \quad (3)$$

$$k = (\gamma, \beta)$$

砂轮渐开线如图 2 所示. 图中 r_{bs} 为渐开线的基圆半径; θ_{os} 为初始角度参数; u_s, θ_{ks} 为渐开线齿面的参数. “ $\pm$ ”号对应渐开线 γ, β 两支.

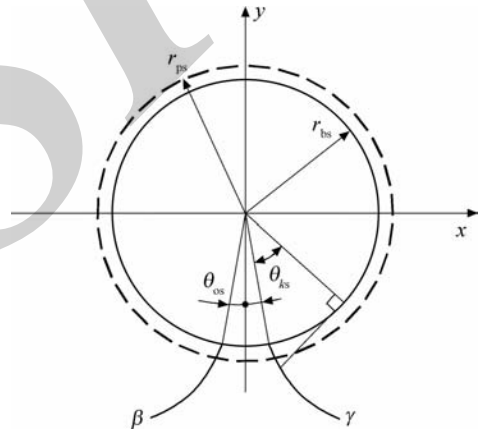


图 2 砂轮渐开线型面

Fig. 2 Involute tooth profile of the grinding wheel

砂轮渐开线单位法向量为

$$\mathbf{n}_s = \frac{\frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s}}{\left| \frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s} \right|} = \begin{bmatrix} -\cos(\theta_{ks} + \theta_{os}) \\ \mp \sin(\theta_{ks} + \theta_{os}) \\ 0 \end{bmatrix} \quad (4)$$

砂轮、面齿轮在动坐标系 $S_s(x_s, y_s, z_s)$ 上啮合点的相对运动速度为

$$\mathbf{v}_s^{(s2)} = \omega_s \begin{bmatrix} -y_s - z_s m_{2s} \cos \varphi_s \\ x_s + z_s m_{2s} \sin \varphi_s \\ m_{2s} (x_s \cos \varphi_s - y_s \sin \varphi_s) \end{bmatrix} \quad (5)$$

式中 m_{2s} 为传动比, $m_{2s} = N_s/N_2 = \omega_2/\omega_s$.

代入啮合方程 $f(u_s, \theta_{ks}, \varphi_s) = \mathbf{n}_s \cdot \mathbf{v}_s^{(s2)} = 0$ 得

$$f(u_s, \theta_{ks}, \varphi_s) =$$

$$r_{bs} - u_s m_{2s} \cos[\varphi_s \pm (\theta_{os} + \theta_{ks})] = 0 \quad (6)$$

则面齿轮齿面方程为

$$\begin{aligned} & \boldsymbol{r}_2(\varphi_s, \theta_{ks}) = \\ & \begin{bmatrix} \pm r_{bs} \left[\cos \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks}) - \frac{\sin \varphi_2}{m_{2s} \cos \xi_{ks}} \right] \\ - r_{bs} \left[\sin \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks}) + \frac{\cos \varphi_2}{m_{2s} \cos \xi_{ks}} \right] \\ - r_{bs} (\cos \xi_{ks} \pm \theta_{ks} \sin \xi_{ks}) \\ 1 \end{bmatrix} \end{aligned} \tag{7}$$

式中 $\xi_{ks} = \varphi_s \pm (\theta_{ks} + \theta_{os})$.

2 面齿轮磨削砂轮修整方法

渐开线碟形砂轮磨削面齿轮为展成磨削, 砂轮截面曲线为渐开线齿廓. 考虑到渐开线齿廓形状外凸的特点, 采用带有修整圆角 R 的金刚滚轮进行修整, 修整时机床轨迹为砂轮截面的渐开线齿廓法向距离为 R 的偏置曲线, 可通过修整圆角 R 部分包络出所需的渐开线, 如图 3 所示.

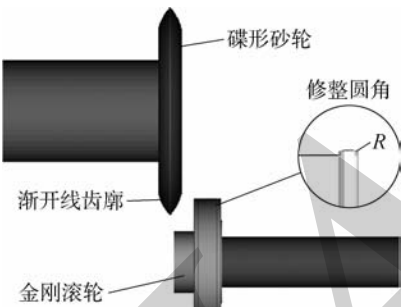


图 3 砂轮修整示意图
Fig. 3 Grinding wheel dressing

渐开线右支 γ 上的点 (x_k, y_k) 对应的偏置距为 R 的偏置曲线上的点为 $(x_{pk}, y_{pk}) = [x_k - R \cos(\theta_s + \theta_k), y_k - R \sin(\theta_s + \theta_k)]$, 如图 4 所示.

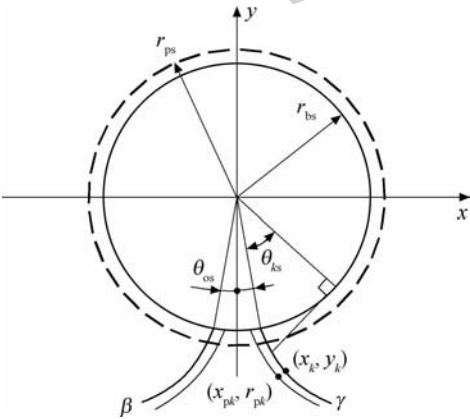


图 4 渐开线偏置曲线示意图
Fig. 4 Offset curve of involute tooth profile

这种修整方法通过 x_1, y, z 三条平动轴的联动即可实现, 故根据某型面齿轮专用磨床的机械结构, 设计其砂轮修整器部分的结构如图 5 所示.

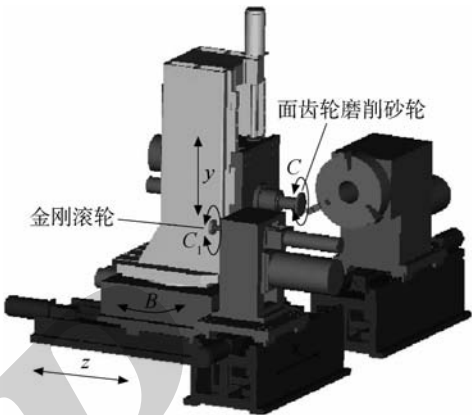


图 5 修整器结构示意图
Fig. 5 Structure of face-gear grinding wheel dresser

运用 VERICUT 软件进行修整加工仿真, 进行仿真结果与理论模型的偏差对比, 验证了修整方法及修整器结构的合理性, 如图 6 所示.

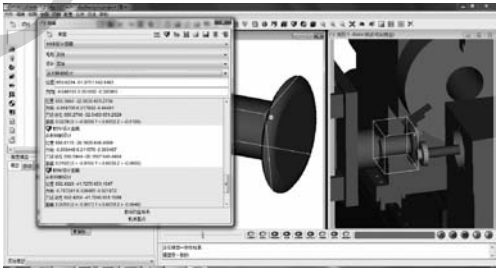


图 6 修整过程加工仿真实验
Fig. 6 Simulation of dressing process

3 砂轮修整误差影响分析

在实际磨削加工时, 砂轮修整过程的误差直接反映到砂轮及面齿轮的型面上. 误差主要来自各轴的对刀误差及运动误差. 分析面齿轮专用砂轮修整器的结构特点, 通过理论推导出带修整误差参数的面齿轮齿面方程, 并通过 MATLAB 生成面齿轮齿面点, 拟合出带误差的面齿轮齿面以及误差分布规律云图, 得到修整误差与面齿轮齿面点误差规律的对应关系, 据此可以根据检测出的齿形误差分布方式反推产生该误差的原因, 为修整过程的误差补偿提供理论依据.

举例说明, 设计面齿轮磨削加工参数如表 1 所示.

表 1 面齿轮设计参数
Table 1 Face gear design parameters

参数	数值	参数	数值
法向模数 m/mm	5	内半径 r_1/mm	100
名义压力角 $\alpha/(\circ)$	20	轴交角 $\gamma/(\circ)$	90
虚拟插齿刀齿数 N_s	20	滚轮圆角半径 R/mm	1
面齿轮齿数 N_2	41	砂轮外半径 r_0/mm	56
外半径 r_2/mm	120		

3.1 B 轴对刀误差

当对刀点位于 $(r_{bs}\sin\theta_{os}, -r_{bs}\cos\theta_{os})$ 时,若 B 轴存在转角误差 $\Delta\theta$,砂轮的渐开线曲线右支将绕点 $(r_{bs}\sin\theta_{os}-R\cos\theta_{os}, -r_{bs}\cos\theta_{os}-R\sin\theta_{os})$ 顺时针旋转 $\Delta\theta$ 角,左支则将绕点 $(-r_{bs}\sin\theta_{os}+R\cos\theta_{os}, -r_{bs}\cos\theta_{os}-R\sin\theta_{os})$ 逆时针旋转 $\Delta\theta$ 角.其中 R 为滚轮圆角半径.设这一点为 (A,B) ,则 B 轴有误差 $\Delta\theta$ 时,砂轮渐开线方程变为

$$\begin{aligned} r_s(u_s, \theta_{ks}) = & \begin{bmatrix} \pm \{r_{bs}\cos\Delta\theta[\sin(\theta_{os} + \theta_{ks}) - \theta_{ks}\cos(\theta_{os} + \theta_{ks})] - \\ r_{bs}\sin\Delta\theta[\cos(\theta_{os} + \theta_{ks}) + \theta_{ks}\sin(\theta_{os} + \theta_{ks})] + A - A \cdot \cos\Delta\theta - B \cdot \sin\Delta\theta\} \\ - r_{bs}\sin\Delta\theta[\sin(\theta_{os} + \theta_{ks}) - \theta_{ks}\cos(\theta_{os} + \theta_{ks}) - \\ r_{bs}\cos\Delta\theta[\cos(\theta_{os} + \theta_{ks}) + \theta_{ks}\sin(\theta_{os} + \theta_{ks})] + B + A \cdot \sin\Delta\theta - B \cdot \cos\Delta\theta \end{bmatrix} \\ & u_s \\ & 1 \\ & k = (\gamma, \beta) \end{aligned} \quad (8)$$

啮合方程为

$$\begin{aligned} f(u_s, \theta_{ks}, \varphi_s) = & r_{bs} - \cos(\theta_{os} + \theta_{ks} - \Delta\theta)(B - B \cdot \cos\Delta\theta + A \cdot \sin\Delta\theta) + \\ & \sin(\theta_{os} + \theta_{ks} - \Delta\theta)(A - A \cdot \cos\Delta\theta - B \cdot \sin\Delta\theta) - u_s m_{2s} \cos[\varphi_s \pm (\theta_{os} + \theta_{ks} - \Delta\theta)] = 0 \end{aligned} \quad (9)$$

坐标变换矩阵不变,而面齿轮方程变为

$$\begin{aligned} r_2(\varphi_s, \theta_{ks}) = & \begin{bmatrix} r_{bs}\cos\varphi_2\cos\Delta\theta(\sin\xi_{ks} \mp \theta_{ks}\cos\xi_{ks}) - r_{bs}\cos\varphi_2\sin\Delta\theta(\theta_{ks}\sin\xi_{ks} \pm \cos\xi_{ks}) \pm \\ (A - A \cdot \cos\Delta\theta - B \cdot \sin\Delta\theta)\cos\varphi_2\cos\varphi_s - (B + A \cdot \sin\Delta\theta - B \cdot \cos\Delta\theta)\cos\varphi_2\sin\varphi_s - u_s\sin\varphi_2 \\ - r_{bs}\sin\varphi_2\cos\Delta\theta(\sin\xi_{ks} \mp \theta_{ks}\cos\xi_{ks}) + r_{bs}\sin\varphi_2\sin\Delta\theta(\theta_{ks}\sin\xi_{ks} \pm \cos\xi_{ks}) \mp \\ (A - A \cdot \cos\Delta\theta - B \cdot \sin\Delta\theta)\sin\varphi_2\cos\varphi_s + (B + A \cdot \sin\Delta\theta - B \cdot \cos\Delta\theta)\sin\varphi_2\sin\varphi_s - u_s\cos\varphi_2 \\ - r_{bs}\cos\Delta\theta(\cos\xi_{ks} \pm \theta_{ks}\sin\xi_{ks}) - r_{bs}\sin\Delta\theta(\sin\xi_{ks} - \theta_{ks}\cos\xi_{ks}) + \\ (A - A \cdot \cos\Delta\theta - B \cdot \sin\Delta\theta)\sin\varphi_s + (B + A \cdot \sin\Delta\theta - B \cdot \cos\Delta\theta)\cos\varphi_s \end{bmatrix} \\ & 1 \end{aligned} \quad (10)$$

式中 $\xi_{ks} = \varphi_s \pm (\theta_{ks} + \theta_{os})$.

当 $\Delta\theta = 0.001 \text{ rad}$ 时,面齿轮齿面点误差云图如图 7 所示.

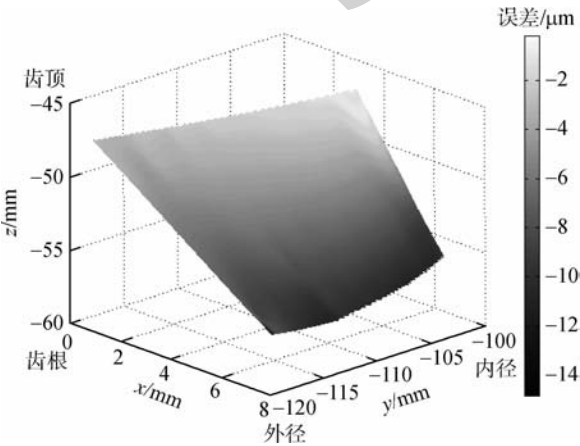


图 7 B 轴对刀误差齿面云图

误差的分布形式为:沿齿宽方向分布均匀,沿齿高方向,齿顶处取最小值为 0,齿根处取最大值.当 $\Delta\theta = \pm 0.001 \text{ rad}$ 时,最大误差为 $\pm 12 \mu\text{m}$ 左右.误差的符号与 $\Delta\theta$ 相反.

齿形误差最大值随 B 轴对刀误差的变化规律如图 8 所示,在正常的误差变动范围内,面齿轮的齿形误差最大值与 B 轴对刀误差的关系基本为线性关系.

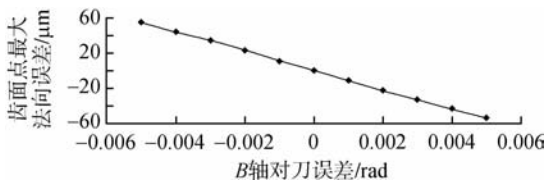


图 8 齿型误差随 B 轴对刀误差的变化规律

Fig. 8 Law between tooth surface error and error of B axis when tool setting

Fig. 7 Error cloud with error of B axis when tool setting

在机床坐标系下,修整过程中存在 B 轴对刀误差的补偿方式为修正程序段中的 $\Delta z' = k\Delta z$, k 的取值与渐开线参数及面齿轮齿面误差实际大小有关.

3.2 z 轴对刀误差

对刀沿 z 轴有误差 Δz 时,砂轮渐开线将沿机床 z 轴方向发生平移,渐开线方程变为

$$\mathbf{r}_s(u_s, \theta_{ks}) = \begin{bmatrix} \pm r_{bs} [\sin(\theta_{os} + \theta_{ks}) - \theta_{ks} \cos(\theta_{os} + \theta_{ks})] \pm \Delta z \cdot \cos \frac{\pi}{N_s} \\ -r_{bs} [\cos(\theta_{os} + \theta_{ks}) + \theta_{ks} \sin(\theta_{os} + \theta_{ks})] + \Delta z \cdot \sin \frac{\pi}{N_s} \\ u_s \\ 1 \end{bmatrix} \quad k = (\gamma, \beta) \quad (11)$$

啮合方程为

$$f(u_s, \theta_{ks}, \varphi_s) = r_{bs} - \Delta z \cdot \sin \left[\frac{\pi}{N_s} \mp (\theta_{os} + \theta_{ks}) \right] - u_s m_{2s} \cos [\varphi_s \pm (\theta_{os} + \theta_{ks})] = 0 \quad (12)$$

坐标变换矩阵不变,而面齿轮方程变为

$$\mathbf{r}_2(\varphi_s, \theta_{ks}) = \begin{bmatrix} r_{bs} [\cos \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks})] - \frac{\sin \varphi_2 \left\{ r_{bs} - \Delta z \cdot \sin \left[\frac{\pi}{N_s} \mp (\theta_{ks} + \theta_{os}) \right] \right\}}{m_{2s} \cos \xi_{ks}} + \\ \Delta z \cdot \cos \varphi_2 \cdot \cos \left(\frac{\pi}{N_s} - \varphi_s \right) \\ -r_{bs} [\sin \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks})] - \frac{\cos \varphi_2 \left\{ r_{bs} - \Delta z \cdot \sin \left[\frac{\pi}{N_s} \mp (\theta_{ks} + \theta_{os}) \right] \right\}}{m_{2s} \cos \xi_{ks}} - \\ \Delta z \cdot \sin \varphi_2 \cdot \cos \left(\frac{\pi}{N_s} + \varphi_s \right) \\ -r_{bs} (\cos \xi_{ks} \pm \theta_{ks} \sin \xi_{ks}) + \Delta z \cdot \sin \left(\frac{\pi}{N_s} + \varphi_s \right) \\ 1 \end{bmatrix} \quad (13)$$

式中 $\xi_{ks} = \varphi_s \pm (\theta_{ks} + \theta_{os})$.

当 $\Delta z = 10 \mu\text{m}$ 时,面齿轮齿面点误差云图如图 9 所示.

误差的分布形式为:沿齿宽方向分布近似均

匀,沿齿高方向,齿顶处取最大值,齿根靠内径处取最小值. 当 $\Delta z = \pm 10 \mu\text{m}$ 时,最小误差为 $\pm 7 \mu\text{m}$,最大误差为 $\pm 10 \mu\text{m}$. 误差符号与 Δz 相同.

在机床坐标系下,修整过程中存在 z 轴对刀误差的补偿方式为修正程序段中的 $z' = z + \Delta z$, Δz 的取值根据面齿轮的齿面误差反推而得.

3.3 x_1 轴对刀误差

对刀沿 x_1 轴有误差 ΔE ,砂轮渐开线将沿机床 x_1 轴方向发生平移,渐开线方程与坐标变换矩阵均发生变化. 砂轮渐开线方程为

$$\mathbf{r}_s(u_s, \theta_{ks}) = \begin{bmatrix} \pm r_{bs} [\sin(\theta_{os} + \theta_{ks}) - \theta_{ks} \cos(\theta_{os} + \theta_{ks})] \mp \Delta E \cdot \sin \frac{\pi}{N_s} \\ -r_{bs} [\cos(\theta_{os} + \theta_{ks}) + \theta_{ks} \sin(\theta_{os} + \theta_{ks})] + \Delta E \cdot \cos \frac{\pi}{N_s} \\ u_s \\ 1 \end{bmatrix} \quad k = (\gamma, \beta) \quad (14)$$

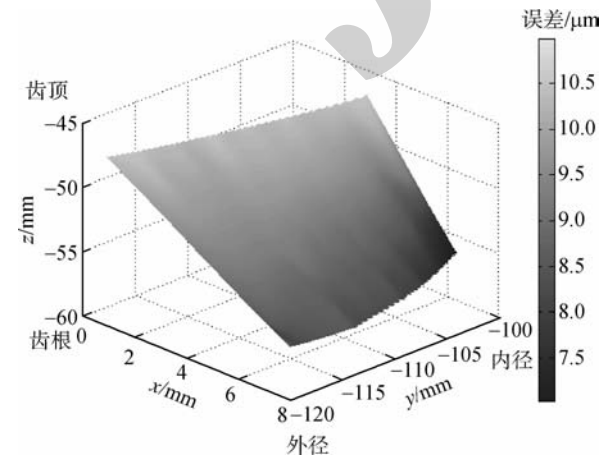


图 9 z 轴对刀误差齿面云图

Fig. 9 Error cloud with error of z axis when tool setting

啮合方程为

$$f(u_s, \theta_{ks}, \varphi_s) = r_{bs} - u_s m_{2s} \cos[\varphi_s \pm (\theta_{os} + \theta_{ks})] + \Delta E \cdot \cos\left[\frac{\pi}{N_s} - (\theta_{os} + \theta_{ks})\right] = 0 \quad (15)$$

面齿轮方程为

$$r_2(\varphi_s, \theta_{ks}) = \begin{bmatrix} r_{bs} [\cos \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks})] \mp \Delta E \cdot \cos \varphi_2 \sin\left(\frac{\pi}{N_s} \pm \varphi_s\right) - u_s \sin \varphi_2 \\ - r_{bs} [\sin \varphi_2 (\sin \xi_{ks} \mp \theta_{ks} \cos \xi_{ks})] + \Delta E \cdot \sin \varphi_2 \sin\left(\varphi_s \pm \frac{\pi}{N_s}\right) - u_s \cos \varphi_2 \\ - r_{bs} (\cos \xi_{ks} \pm \theta_{ks} \sin \xi_{ks}) + \Delta E \cdot \cos\left(\frac{\pi}{N_s} \pm \varphi_s\right) \end{bmatrix} \quad (16)$$

式中

$$\xi_{ks} = \varphi_s \pm (\theta_{ks} + \theta_{os})$$

$$u_s = \frac{r_{bs} + \Delta E \cdot \cos\left[\frac{\pi}{N_s} - (\theta_{os} + \theta_{ks})\right]}{m_{2s} \cos[\varphi_s \pm (\theta_{os} + \theta_{ks})]}$$

当 $\Delta E = 10 \mu\text{m}$ 时, 齿面点误差云图如图 10 所示。

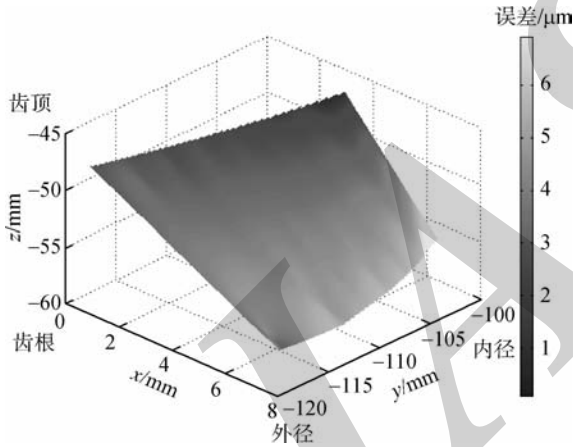


图 10 x_1 轴对刀误差齿面云图

Fig. 10 Error cloud with error of x_1 axis when tool setting

误差的分布形式为: 齿宽方向接近均匀分布, 齿高方向由齿顶向齿根逐渐变大, 靠外径处变化较平缓, 靠内径处变化较剧烈, 齿根靠内径处误差取得最大值。当 $\Delta E = \pm 10 \mu\text{m}$ 时, 最大误差为 $\pm 7 \mu\text{m}$ 左右, 误差的符号与 ΔE 相同。

齿形误差的最大值随 z 轴及 x_1 轴对刀误差变化的规律如图 11 所示。在正常的误差变动范围内, 面齿轮的齿面精度与对刀误差的关系基本为线性关系, 且对 z 轴对刀误差更为敏感。

在机床坐标系下, 修整过程中存在 x_1 轴对刀误差的补偿方式为修正程序段中的 $x' = x + \Delta x$, Δx 的取值根据面齿轮的齿面误差反推而得。

3.4 y 轴对刀误差

对刀沿 y 轴有误差 $\pm \Delta y$ 时, 实际进行修整的点为两圆相切点。如图 12 所示。

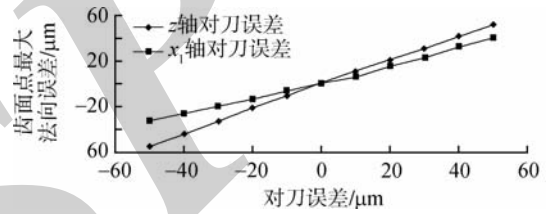


图 11 齿面点误差随 z, x_1 轴对刀误差的变化规律

Fig. 11 Law between tooth surface error and error of z axis, x_1 axis when tool setting

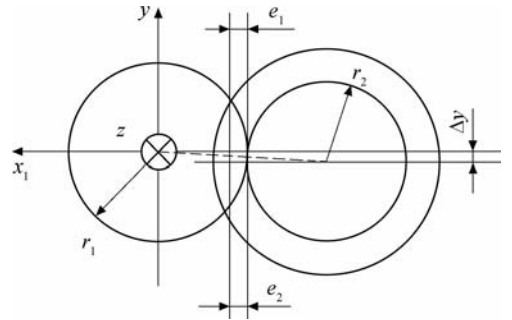


图 12 y 轴对刀误差分析

Fig. 12 Error of y axis when tool setting

当刀位点沿偏移后的 x_1 轴移动 e_1 时, 实际在砂轮上体现的变化量为 $e_2 = \sqrt{e_1^2 + r_2^2} + 2e_1 r_2 \cos \theta - r_2$, 其中 $\theta = \arcsin \frac{\Delta y}{r_1 + r_2}$, r_1 为滚轮半径, r_2 为砂轮渐开线根部所在的圆半径。则误差量 Δe 是一个与滚轮半径、砂轮半径相关的量。当取常用值 $r_1 = 50 \text{ mm}$, $r_2 = 50 \text{ mm}$, 程序单行走刀量 $e_1 = 0.1 \text{ mm}$, y 向误差 $\Delta y = 0.1 \text{ mm}$ 时, $\Delta e = -4.99 \times 10^{-5} \mu\text{m}$ 。故得出结论 y 轴对刀误差为不敏感项, 是一个可以忽略的误差项。

3.5 x_1 轴运动误差

当对刀点位于 $(r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 时, 若 x_1 轴存在运动误差, 则砂轮的渐开线曲线(右支)将绕点 $(r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 发生扭剪变形, 左支则将绕点 $(-r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 发生扭剪变形。

以右支为研究对象, 设点 $(\cos \frac{\pi}{N_s} r_{bs} \sin \theta_{os} -$

$$\mathbf{r}_s(u_s, \theta_{ks}) = \begin{bmatrix} \left(\cos^2 \frac{\pi}{N_s} + k_x \sin^2 \frac{\pi}{N_s} \right) x + \left(\frac{1-k_x}{2} \cdot \sin \frac{2\pi}{N_s} \right) y - (1-k_x) \sin \frac{\pi}{N_s} \cdot B \\ \left(\sin^2 \frac{\pi}{N_s} + k_x \cos^2 \frac{\pi}{N_s} \right) y + \left(\frac{1-k_x}{2} \cdot \sin \frac{2\pi}{N_s} \right) x + (1-k_x) \cos \frac{\pi}{N_s} \cdot B \\ u_s \\ 1 \end{bmatrix} \quad (17)$$

设其为 $\mathbf{r}_s(u_s, \theta_{ks}) = [x_s \ y_s \ u_s \ 1]^T$ 。

啮合方程中的渐开线单位法向量 $\mathbf{n}_s$ 为

$$\mathbf{n}_s = \frac{\frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s}}{\left| \frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s} \right|} = \begin{bmatrix} \pm \sin \frac{\pi}{N_s} \sin \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) - k_x \cos \frac{\pi}{N_s} \cos \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) \\ \sqrt{\sin^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) + k_x^2 \cos^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right)} \\ \mp \cos \frac{\pi}{N_s} \sin \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) - k_x \sin \frac{\pi}{N_s} \cos \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) \\ \sqrt{\sin^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right) + k_x^2 \cos^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s} \right)} \\ 0 \end{bmatrix} \quad (18)$$

设其为 $\mathbf{n}_s = [n_a \ n_b \ 0]^T$ 。

相对运动速度 $v_s^{(s2)}$ 不变, 啮合方程为

$$u_s = \frac{n_a y_s - n_b x_s}{m_{2s} (n_b \sin \varphi_s - n_a \cos \varphi_s)}$$

坐标变换矩阵不变, 而面齿轮方程变为

$$\mathbf{r}_2(\varphi_s, \theta_{ks}) = \begin{bmatrix} \cos \varphi_2 \cos \varphi_s x_s - \cos \varphi_2 \sin \varphi_s y_s - \sin \varphi_2 u_s \\ -\sin \varphi_2 \cos \varphi_s x_s + \sin \varphi_2 \sin \varphi_s y_s - \cos \varphi_2 u_s \\ \sin \varphi_s x_s + \cos \varphi_s y_s \\ 1 \end{bmatrix} \quad (19)$$

当 x_1 轴运动误差系数 $k_x = 1.001$ 时, 面齿轮齿面点误差云图如图 13 所示。

误差的分布形式为: 沿半径方向分布均匀, 沿齿高方向, 齿顶处取最小值 0, 逐渐变化至齿根处取最大值。当 $k_x - 1 = \pm 0.001$ 时, 最大误差为 $\pm 6 \mu\text{m}$ 左右。误差符号与 $k_x - 1$ 符号相反。

3.6 z 轴运动误差

当对刀点位于 $(r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 时, 若 z 轴存在运动误差, 则砂轮的渐开线曲线(右支)将绕点 $(r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 发生扭剪变形, 左支则

$\sin \frac{\pi}{N_s} r_{bs} \cos \theta_{os}, -\sin \frac{\pi}{N_s} r_{bs} \sin \theta_{os} - \cos \frac{\pi}{N_s} \cos \theta_{os})$

为 (A, B) , 若 x_1 轴运动误差系数为 k_x , 即 x_1 向实际运动距离为理论值的 k_x 倍时, 砂轮渐开线右支方程变为

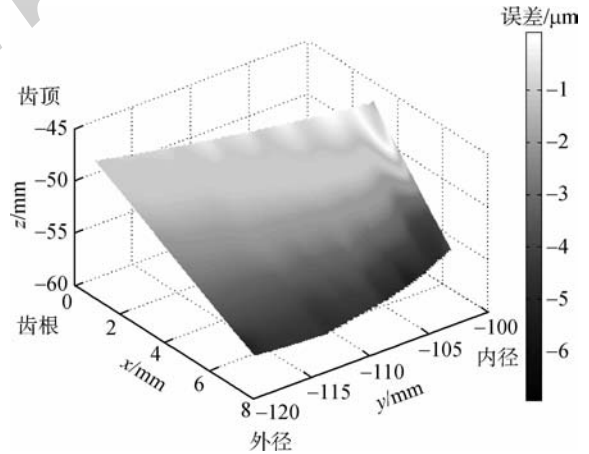


图 13 x_1 轴运动误差齿面云图

Fig. 13 Error cloud with kinematic error of x_1 axis

将绕点 $(-r_{bs} \sin \theta_{os}, -r_{bs} \cos \theta_{os})$ 发生扭剪变形。以右支为研究对象, 设点 $(\cos \frac{\pi}{N_s} r_{bs} \sin \theta_{os} - \sin \frac{\pi}{N_s} r_{bs} \cos \theta_{os}, -\sin \frac{\pi}{N_s} r_{bs} \sin \theta_{os} - \cos \frac{\pi}{N_s} \cos \theta_{os})$ 为 (A, B) , 若 z 轴运动误差系数为 k_z , 即 z 向实际运动距离为理论值的 k_z 倍时, 砂轮渐开线右支方程变为

$$\mathbf{r}_s(u_s, \theta_{ks}) = \begin{bmatrix} \left(k_z \cos^2 \frac{\pi}{N_s} + \sin^2 \frac{\pi}{N_s}\right)x + \left(\frac{k_z - 1}{2} \cdot \sin \frac{2\pi}{N_s}\right)y - (k_z - 1) \cos \frac{\pi}{N_s} \cdot A \\ k_z \sin^2 \frac{\pi}{N_s} + \cos^2 \frac{\pi}{N_s} y + \left(\frac{k_z - 1}{2} \cdot \sin \frac{2\pi}{N_s}\right)x - (k_z - 1) \sin \frac{\pi}{N_s} \cdot A \\ u_s \\ 1 \end{bmatrix} \quad (20)$$

设其为 $\mathbf{r}_s(u_s, \theta_{ks}) = [x_s \ y_s \ u_s \ 1]^T$.

渐开线单位法向量为

$$\mathbf{n}_s = \frac{\frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s}}{\left| \frac{\partial \mathbf{r}_s}{\partial \theta_{ks}} \cdot \frac{\partial \mathbf{r}_s}{\partial u_s} \right|} = \begin{bmatrix} \pm k_z \sin \frac{\pi}{N_s} \sin \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) - \cos \frac{\pi}{N_s} \cos \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) \\ \sqrt{k_z^2 \sin^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) + \cos^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right)} \\ \mp k_z \cos \frac{\pi}{N_s} \sin \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) - \sin \frac{\pi}{N_s} \cos \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) \\ \sqrt{k_z^2 \sin^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right) + \cos^2 \left(\theta_{os} + \theta_{ks} \mp \frac{\pi}{N_s}\right)} \\ 0 \end{bmatrix} \quad (21)$$

设其为 $\mathbf{n}_s = [n_a \ n_b \ 0]^T$.

相对运动速度 $\mathbf{v}_s^{(s2)}$ 不变,啮合方程为

$$u_s = \frac{n_a y_s - n_b x_s}{m_{2s} (n_b \sin \varphi_s - n_a \cos \varphi_s)} \quad (22)$$

坐标变换矩阵不变,而面齿轮方程变为

$$\mathbf{r}_2(\varphi_s, \theta_{ks}) = \begin{bmatrix} \cos \varphi_2 \cos \varphi_s x_s - \cos \varphi_2 \sin \varphi_s y_s - \sin \varphi_2 u_s \\ -\sin \varphi_2 \cos \varphi_s x_s + \sin \varphi_2 \sin \varphi_s y_s - \cos \varphi_2 u_s \\ \sin \varphi_s x_s + \cos \varphi_s y_s \\ 1 \end{bmatrix} \quad (23)$$

当 z 向运动误差系数 $k_z = 1.001$ 时,面齿轮齿面点误差云图如图 14 所示.

误差的分布形式为:沿半径方向分布均匀,沿齿高方向,齿顶处取最小值 0,逐渐变化至齿根处

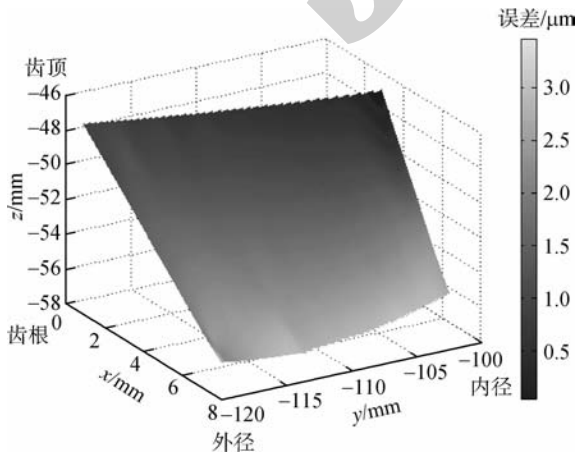


图 14 z 轴运动误差齿面云图

Fig. 14 Error cloud with kinematic error of z axis

取最大值.当 $k_z - 1 = \pm 0.001$ 时,最大误差为 $\pm 3 \mu\text{m}$ 左右.误差符号与 $k_z - 1$ 符号相同.

3.7 小结

仿真结果表明,在修整对刀误差方面,面齿轮的齿面精度对 z 向对刀误差及 B 向对刀误差更为敏感,对 x_1 向误差敏感程度较低,在常用的机床参数范围下, y 向误差对齿面精度的影响可以忽略;在修整过程机床运动误差方面,面齿轮的齿面精度对 x_1 轴的运动误差更为敏感.

4 加工实验及修整误差补偿

按上文所述参数进行面齿轮数控磨削加工及修整试验,如图 15 所示.



图 15 面齿轮修整及磨削加工试验

Fig. 15 Dressing and grinding experiment of face-gear

在 Xorbit-87 桥式三坐标测量机上进行加工样件检测,如图 16 所示.

检测区域为距齿顶、齿根、内外径各 10% 的区域,检测点数取齿高方向 5 个点,径向 9 个点,共计 45 个点.测点与理论值的法向误差如图 17 所示.

分析检测数据可知,齿面误差是砂轮型面误差与磨削误差综合作用的结果.误差分布方式为:整个齿面为负误差,沿齿高方向从齿顶到齿根误



图 16 面齿轮检测试验

Fig. 16 Measurement experiment of face-gear

差绝对值逐渐增大,由砂轮修整误差引起的齿顶处最小负误差接近 0,沿齿宽方向从外端到内端误差绝对值逐渐增大,最大负误差出现在齿根靠内径处,为 $-135.47\mu\text{m}$,将齿面点误差分布规律及误差最大值与理论推导的数据对比可知,误差产生的原因可能是修整过程中存在 B 向对刀误差 $\Delta\theta=0.004\text{ rad}$,据此进行修整误差补偿,将砂轮修整程序中相邻刀位点之间的 Δz 值修正为 $\Delta z'=1.02\Delta z$. 补偿后再次加工与检测,得测点数据如图 18 所示.

补偿后齿面点沿齿高方向的误差变化基本消除,沿径向的误差需在磨削程序中调整参数进行补偿. 补偿后最大负误差为 $-93.29\mu\text{m}$,最大误差值减小了 31.14%,齿面精度得到了提高,验证了修整误差分析结果的正确性.

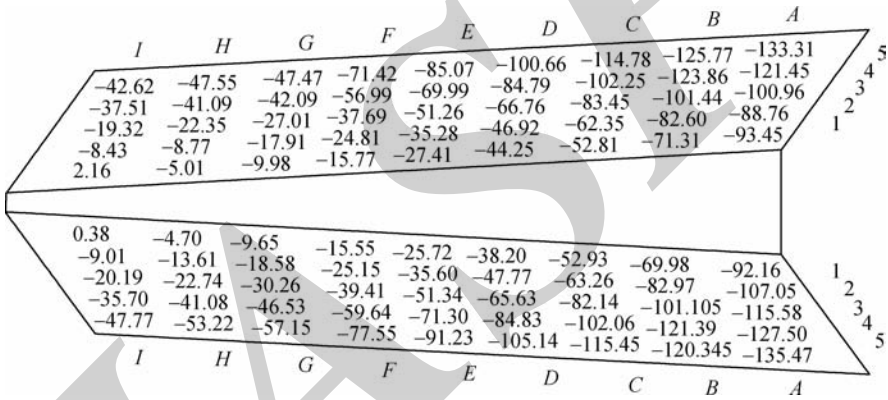


图 17 初试样件检测点法向误差

Fig. 17 Normal error of measuring points of face-gear

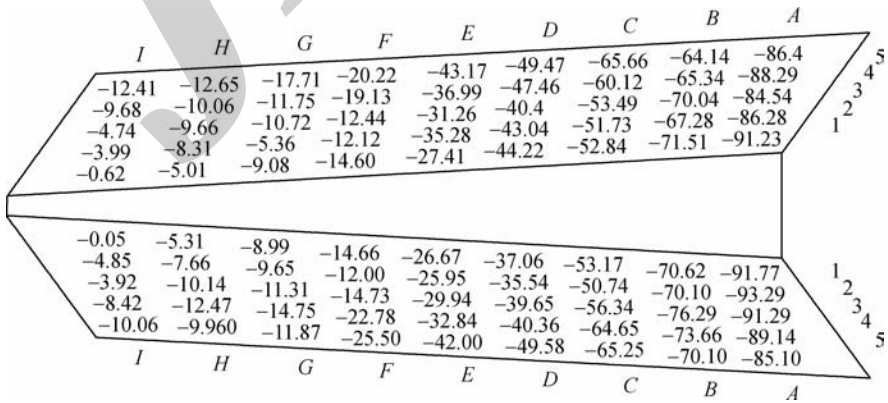


图 18 误差补偿后样件检测点法向误差

Fig. 18 Normal error of measuring points of face-gear after error compensation

5 结 论

1) 针对面齿轮磨削专用渐开线碟形砂轮的型面特点,提出一种偏置曲线修整方法.

2) 以某型面齿轮专用磨床为基础,进行了砂轮修整器的结构设计,通过 VERICUT 软件加工仿真验证了修整器结构及修整方法的合理性.

3) 推导了存在各类修整误差情况下的面齿

轮齿面方程,运用 MATLAB 生成了齿面误差云图,分析了误差对面齿轮型面的影响规律,总结了面齿轮型面精度对各类修整误差的敏感程度,为误差补偿提供了理论依据。

4) 通过面齿轮磨齿、修整与检测实验,根据误差分析结果进行了修整过程的误差补偿,减小了齿面偏差,提高了齿面精度。

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